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Quasi-gravity in Branes[1]

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In contrast with pseudo-gravitational effects that are mathematically analogous but physically quite distinct from gravity, this presentation deals with a kind of quasi-gravitational effect that can act in an asymmetrically moving brane worldsheet in a manner that approximates (and in a crude analysis might be physically indistinguishable from) the effect that would arise from genuine gravitation, of ordinary Newtonian type in nonrelativistic applications and of scalar–tensor (Jordan–Brans–Dicke rather than pure Einstein) type in relativistic applications.

KEY WORDS: brane; gravity; worldsheet.

1. INTRODUCTION

In order to avoid confusion at the outset, it is to be emphasized that the concept of quasi-gravity—in the sense used here—is quite distinct from that of pseudo-gravity of the kind considered by authors such as Unruh (1995), Visser (1998), and Volovik (2001). That kind of pseudo-gravity involves effects whose mathematical description is more or less analogous to that of gravity, but whose physical nature is quite distinct. Such effects typically involve another Lorentz signature metric that coexists with the ordinary space–time metric (whose deviations from flatness are interpretable as corresponding to true gravity) but couples to matter in an entirely different way, specifying pseudo-light cones that typically govern the propagation not of real light, nor gravity, but of some quite independent excitation such as sound.

The purpose of the present contribution is to draw attention to something rather different, what may be described as quasi-gravitational effects, meaning phenomena that affect matter locally in approximately the same physical manner as true gravity, even though their origin and detailed behavior may be rather different. In the context of nonrelativistic Newtonian gravitation theory, the most familiar example of such a quasi-gravitational effect is the centrifugal field attributable to

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the rotation of the earth that modifies the locally observable Galilean acceleration field by contributing a term that is to be added to the strictly gravitational contribution due to the Newtonian inverse square law attraction due to the terrestrial matter distribution. Although this centrifugal quasi-gravitational contribution is indistinguishable from the truly gravitational contribution in a crude laboratory experiment, the difference is of course detectable, via the Coriolis effect, in more sensitive experiments such as that of the Foucault pendulum.

The kind of quasi-gravitational effect I wish to describe here is something that modifies the induced space–time metric on the $(q+1)$ dimensional worldsheet of a q-brane in a higher dimensional background in a manner that approximates the effect of true $(q+1)$-dimensional gravity, even though its origin and precise nature is essentially different. It is of particular potential interest in the currently fashionable context of models that represent our four-dimensional universe as a 3-brane in a five-dimensional background, though in the kind of scenarios that are most commonly envisaged the effect considered here would be excluded by the usual assumption of symmetry between the two opposite sides of the 3-brane, while even if the symmetry assumption were dropped [as has recently been proposed in cases where the q-brane worldsheet is coupled to a background gauge $(q+1)$ form (Battye and Carter, 2001; Carter and Uzan, 2001; Kehagias and Kiritsis, 1999)] the quasi-gravitational effect (Carter *et al.*, 2001) could still be overwhelmed by much stronger effects of genuinely gravitational origin (just as the terrestrial centrifugal effect is overwhelmed by the centrally directed genuinely gravitational attraction).

2. EQUATION OF MOTION OF BRANE WORLDSHEET

The effect to be considered here is derivable directly by perturbing the general-purpose brane worldsheet equation of motion, which is given (Carter, 1995) in terms of the second fundamental tensor $K_{\mu\nu}^{\ \ \rho}$ of the brane worldsheet and of the corresponding worldsheet stress-energy density tensor $\overline{T}^{\mu\nu}$ of the brane by

$$\overline{T}^{\mu\nu} K_{\mu\nu}^{\ \ \rho} = \perp_{\mu}^{\rho} \ \overline{f}^{\mu}, \tag{1}$$

where $\overline{f}^{\mu}$ is the external force density, if any, acting on the brane, and $\perp_{\mu}^{\rho}$ is the orthogonal projection tensor. The complementary (rank $p+1$) tangential projection tensor

$$\gamma_{\nu}^{\mu} = g_{\nu}^{\mu} - \perp_{\nu}^{\mu}, \tag{2}$$

i.e. the first fundamental tensor, defines the tangential covariant differentiation operator

$$\overline{\nabla}_{\mu} = \gamma_{\mu}^{\nu} \nabla_{\nu}, \tag{3}$$

whose action on the first fundamental tensor defines the second fundamental tensor according to the specification

$$K_{\mu\nu}^{\ \ \rho} = \gamma_\nu^{\ \sigma} \overline{\nabla}_\mu \gamma_\sigma^{\ \rho}, \tag{4}$$

which is such so as to ensure the Weingarten symmetry condition

$$K_{\mu\nu}^{\ \ \rho} = K_{\nu\mu}^{\ \ \rho}, \tag{5}$$

as a worldsheet integrability condition, as well as having the more obvious tangentiality and orthogonality properties

$$K_{\mu\nu}^{\ \ \sigma} \gamma_\sigma^{\ \rho} = 0 = \perp_\mu^{\lambda} K_{\lambda\nu}^{\ \ \rho}, \tag{6}$$

while its trace

$$K^\rho = K_\mu^{\ \mu\rho} = \overline{\nabla}_\nu \gamma^{\nu\rho} \tag{7}$$

inherits the simple worldsheet orthogonality property

$$\gamma_{\ \sigma}^{\rho} K^\sigma = 0. \tag{8}$$

3. PERTURBED WORLDSHEET CONFIGURATION

The quasi-gravity effect to be considered occurs (in its simplest form) when the total surface stress-energy tensor $\overline{T}^{\mu\nu}$ is dominated by an isotropic (Dirac–Nambu–Goto type) contribution specified by a large fixed tension, say T_∞, together with a small additional contribution $\tau^{\mu\nu}$ arising from the effect of local fields on the brane (representing the observable matter of the universe in brane-world scenarios) in the form

$$\overline{T}^{\mu\nu} = -T_\infty \gamma^{\mu\nu} + \tau^{\mu\nu}, \tag{9}$$

in the presence of an external force of the commonly occurring kind (including a Magnus force on a string and a wind force on a sail) that is automatically orthogonal to the worldsheet $\gamma_\nu^{\mu} \overline{f}^\nu = 0$ so that the orthogonal projection on the right-hand side in (1) is superfluous. Then if the observable matter contribution $\tau^{\mu\nu}$ were absent, the dynamical equation of motion (1) would reduce to the simple form

$$T_\infty K^\rho = \overline{f}^\rho. \tag{10}$$

Starting from an almost uniform (low curvature) reference configuration of this kind, one can consider an actual configuration that deviates from this because of the presence of a matter distribution $\tau^{\mu\nu}$ confined within a lengthscale that is relatively small (compared with the reference curvature scale) for which the dominant terms in the dynamical equation obtained by perturbation of (1) can be seen to be given by an expression of the form

$$T_\infty \delta K^\rho = \tau^{\mu\nu} K_{\mu\nu}^{\ \ \rho}, \tag{11}$$

in which the perturbation δK^ρ of the curvature is given in terms of the Dalembertian wave operator $\overline{\Box} = \overline{\nabla}^\nu \overline{\nabla}_\nu$ of the $(q+1)$ dimensional worldsheet metric and of the surface orthogonal vector field ξ^μ specifying the displacement of the worldsheet, by an expression of the form

$$\delta K^\rho \simeq \overline{\Box}\xi^\rho, \tag{12}$$

which is obtained from the general curvature perturbation formula (Carter, 1995) by retaining only the gradient terms of highest order, which are the ones that dominate in the localized (short lengthscale) limit.

For a brane worldsheet matter distribution that is approximately specified with respect to the relevant tangent rest frame unit vector $\overline{u}^\mu$ ($\overline{u}^\nu \overline{u}_\nu = -1$) by a stress-energy density tensor of the nonrelativistic form

$$\tau^{\mu\nu} \simeq \overline{\rho}\overline{u}^\mu \overline{u}^\nu, \tag{13}$$

in terms of a surface mass density $\overline{\rho}$ whose space section volume integal determines the corresponding total mass say, M, the resulting equation takes the form

$$T_\infty \overline{\Box}\xi^\rho \simeq \overline{\rho}\overline{u}^\mu \overline{u}^\nu K_{\mu\nu}^{\ \ \rho}, \tag{14}$$

in which owing to the staticity the (hyperbolic) Dalembertian operator will reduce to a Laplacian operator (of elliptic type), so that for a $(q-2)$ spherically symmetric distribution the solution will be expressible in terms of the radial distance r from the center by an expression that for $q \geq 3$ will have the power law form

$$T_\infty \xi^\mu = -\frac{M}{(\mathrm{q}-2)\Omega^{[q-1]} r^{q-2}} a^\mu, \tag{15}$$

in terms of the rest frame orthogonal worldsheet acceleration vector $a^\mu = \overline{u}^\nu \overline{\nabla}_\nu \overline{u}^\mu$ given by

$$a^\rho = \overline{u}^\mu \overline{u}^\nu K_{\mu\nu}^{\ \ \rho}. \tag{16}$$

For the familiar, experimentally accessible, case of an ordinary membrane, with $q = 2$, there will be an analogous formula involving radial dependence of logarithmic rather than power law type.

3.1. Quasi-gravitational Metric Perturbations

Under the conditions described in the preceeding section, the brane worldsheet geometry characterized by the fundamental tensor $\gamma_{\mu\nu}$ will be subject to a corresponding perturbation $\overline{h}_{\mu\nu} = \delta_{\gamma\mu\nu}$, which will be given (Carter, 1995) in terms of the second fundamental tensor of the unperturbed reference state by an expression of the form

$$\overline{h}_{\mu\nu} = -K_{\mu\nu}^{\ \ \rho}\xi_\rho. \tag{17}$$

This perturbation will have a time component

$$\bar{h}_{00} = \bar{u}^{\mu}\bar{u}^{\nu}\bar{h}_{\mu\nu} \tag{18}$$

given by the formula

$$\bar{h}_{00} = -2a^{\rho}\xi_{\rho}, \tag{19}$$

while the trace $\bar{h}^{\nu}_{\nu}$ will be given by an expression of the analogous form

$$\bar{h}^{\nu}_{\nu} = -2K^{\rho}\xi_{\rho}, \tag{20}$$

Evaluating (19) explicitly using (37), one sees that it is reducible to an expression of the standard (dimensionally generalized; Arkani-Hamed *et al.*, 1999) Newtonian form

$$\bar{h}_{00} = \frac{2\mathrm{G}_{[q+1]}M}{r^{q-2}}, \tag{21}$$

with the relevant generalised Newton constant given by

$$\mathrm{G}_{[q+1]} = \frac{1}{(q-2)\Omega^{[q-1]}T_{\infty}} a^{\rho}a_{\rho}. \tag{22}$$

Although this mechanism will thus effectively simulate Newtonian-type gravitational attraction in so far as its effect on nonrelativistic Keppler type orbits is concerned, it leads to a value for the ratio $\bar{h}_{00}/\bar{h}^{\nu}_{\nu}$ that can be seen from (19) and (20) to be given by (15) as

$$\frac{\bar{h}_{00}}{\bar{h}^{\nu}_{\nu}} = \frac{a^{\rho}a_{\rho}}{a^{\nu}K_{\nu}}, \tag{23}$$

which will not in general agree with the prediction of Einstein's theory. Although the ensuing prediction for the relativistic behavior (e.g. of light deflection) will thereby deviate from that of Einstein's purely tensorial theory of gravity, it gives a result that will be shown to be matchable by a more general theory of the Jordan–Brans–Dicke type to be described in the next section.

4. JORDAN–BRANS–DICKE TYPE THEORIES

The action integral

$$\mathcal{I} = \int \mathcal{L}\|g\|^{1/2} d^{(q+1)}x, \quad \mathcal{L} = \mathcal{L}_{\mathrm{D}} + \mathcal{L}_{\mathrm{M}}, \tag{24}$$

for a Jordan–Bran–Dicke-type scalar–tensor theory (Dicke, 1964) in a $(q+1)$ dimensional space–time, with metric $g_{\mu\nu}$ in a Dicke-type conformal gauge (meaning one in which the weak equivalence principle is satisfied), is given by a Lagrangian density consisting of a Dicke-type gravitational contribution $\mathcal{L}_{\mathrm{D}}$ involving a dilatonic scalar field Φ as well as the metric, and an ordinary matter contribution $\mathcal{L}_{\mathrm{M}}$

that is independant of Φ, with the Dicke contribution given in terms of a coupling constant $\omega_{\rm D}$ by an expression of the form

$$\mathcal{L}_{\rm D} = \frac{1}{2(q-1)\Omega^{[q-1]}}\left(\Phi R - \frac{\omega_{\rm D}}{\Phi} g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}\right), \tag{25}$$

where R is the Ricci scalar for the metric $g_{\mu\nu}$ and $\Omega^{[q-1]}$ is the surface area of the unit $(q-1)$ sphere, which, for an ordinary four-dimensional space–time, with space dimension $q=3$, will be given by $\Omega^{[2]} = 4\pi$.

In terms of the trace of the material stress energy density tensor

$$T_{\rm M}^{\mu\nu} = 2\frac{\partial \mathcal{L}_{\rm M}}{g_{\mu\nu}} - \mathcal{L}_{\rm M} g^{\mu\nu}, \tag{26}$$

the scalar wave equation for such a theory will be given in terms of the Dalembertian operator $\Box = \nabla_\nu \nabla^\nu$ by

$$\Box\Phi = \alpha_{\rm D}\Omega^{[q-1]} T^{\nu}_{{\rm M}\nu}, \tag{27}$$

in terms of a dilatonic coupling constant $\alpha_{\rm D}$, which is given in terms of the original Dicke constant $\omega_{\rm D}$ by

$$\frac{1}{\alpha_{\rm D}} = \omega_{\rm D} + \frac{\rm q}{\rm q - 1}. \tag{28}$$

To deal with the gravitational equations, it is convenient to express the dilatonic amplitude Φ in terms of some fixed value $\hat\Phi$ and of a dimensionless scalar field ϕ in the form

$$\Phi = {\rm e}^{-2\phi}\hat\Phi \tag{29}$$

and to change to what is known as an Einstein gauge by a conformal transformation $g_{\mu\nu} \mapsto \hat g_{\mu\nu}$ that is specified by setting

$$g_{\mu\nu} = {\rm e}^{2\sigma}\hat g_{\mu\nu}, \tag{30}$$

where the field σ is given in terms of ϕ by by the proportionality relation

$$2\phi = (q-1)\sigma. \tag{31}$$

In terms of the Einstein type conformal gauge the action (24) will take the form

$$\mathcal{I} = \int \widehat{\mathcal{L}} \|\widehat{g}\|^{1/2}\, d^{(q+1)}x, \quad \widehat{\mathcal{L}} = \widehat{\mathcal{L}}_{\rm D} + \widehat{\mathcal{L}}_{\rm L}, \tag{32}$$

with the matter contribution given by

$$\widehat{\mathcal{L}}_{\rm M} = e^{(q+1)\sigma}\mathcal{L}_{\rm M}, \tag{33}$$

while $\widehat{\mathcal{L}}_{\rm D}$ is given as the sum of an ordinary Einstein–Hilbert-type term and a linear scalar field contribution in the form

$$\widehat{\mathcal{L}}_{\rm D} = \frac{\widehat{\Phi}}{2(q-1)\Omega^{[q-1]}}\left(\widehat{R} - \frac{4}{\alpha_{\rm D}}\widehat{g}^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right), \tag{34}$$

where $\widehat{R}$ is the Ricci scalar for the Einstein metric $\widehat{g}_{\mu\nu}$ and $\alpha_{\rm D}$ is the constant given by (28), while the fixed amplitude $\widehat{\Phi}$ now acts as the inverse of the (dimensionally generalised) Newton constant, which can be identified as

$$\widehat{\mathrm{G}}_{[q+1]} = \frac{1}{\widehat{\Phi}}. \tag{35}$$

In this reformulation there will be a matter stress energy density contribution given by

$$\widehat{T}^{\mu}_{\mathrm{M}\nu} = e^{(q+1)\sigma}\, T^{\mu}_{\mathrm{M}\nu} \tag{36}$$

whose trace will act as the source for the linear wave equation for ϕ, which will be expressible in the form

$$\widehat{\Box}\phi = -\frac{1}{2}\Omega^{[q-1]}\widehat{\mathrm{G}}_{[q+1]}\alpha_{\rm D}\widehat{T}^{\rho}_{\mathrm{M}\rho}, \tag{37}$$

where $\widehat{\Box}$ is the Dalembertian operator for the Einstein metric $\widehat{g}_{\mu\nu}$. The corresponding Einstein-type gravitational equations will be expressible as

$$\widehat{R}_{\mu\nu} - \frac{1}{2}\widehat{R}\,\widehat{g}_{\mu\nu} = \frac{2}{\alpha_{\rm D}}(2\phi_{\mu}\phi_{\nu} - \widehat{g}_{\mu\nu}\widehat{g}^{\rho\sigma}\phi_{\rho}\phi_{\sigma}) + (q-1)\Omega^{[q-1]}\widehat{\mathrm{G}}_{[q+1]}\widehat{T}_{\mathrm{M}\mu\nu}. \tag{38}$$

5. LINEARIZED LOCAL SCALAR–TENSOR FIELD CONFIGURATIONS

Let us now consider the weak field, low source density, limit in which the system can be linearized with respect to the dilatonic perturbation field ϕ and the Einstein metric perturbation field $\widehat{h}_{\mu\nu}$ defined relative to a flat Minkowski background metric $\eta_{\mu\nu}$ by setting

$$\widehat{g}_{\mu\nu} = \eta_{\mu\nu} + \widehat{h}_{\mu\nu}. \tag{39}$$

Equation (37) for ϕ is already linear as it stands, while the corresponding linearized Einstein equation for $\widehat{h}_{\mu\nu}$ is obtainable from (38) in the standard form

$$\Box\widehat{h}_{\mu\nu} = -2\Omega^{[q-1]}\widehat{\mathrm{G}}_{[q+1]}\big((q-1)\widehat{T}_{\mathrm{M}\mu\nu} - \widehat{T}^{\rho}_{\mathrm{M}\rho}\eta_{\mu\nu}\big). \tag{40}$$

The gravitational field that is directly measured by the observation of Keppler-type orbits will not be given by this Einstein-type metric $\widehat{g}_{\mu\nu}$ (to which, owing to the involvement of ϕ in the relevant stress energy tensor $\widehat{T}^{\mu\nu}_{\mathrm{M}}$, the usual equivalence principle does not apply) but by the original Dicke-type metric $g_{\mu\nu}$, which will be expressible analogously to (39) by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{41}$$

with

$$h_{\mu\nu} = \widehat{h}_{\mu\nu} + 2\sigma\eta_{\mu\nu}, \tag{42}$$

to linear order, by (30). Usng the relation (11), it can be seen, by combining the wave equations (37) and (40), that the directly observable metric perturbation $h_{\mu\nu}$ will be given, to linear order, by

$$\Box h_{\mu\nu} = -2\Omega^{[\mathrm{q}-1]}\widehat{\mathrm{G}}_{[q+1]}\big((q-1)T_{\mathrm{M}\mu\nu} + (1-\Delta_{\mathrm{D}})T^{\rho}_{\mathrm{M}\rho}\eta_{\mu\nu}\big), \tag{43}$$

in which the dilatonic deviation constant is given by

$$\Delta_{\mathrm{D}} = \frac{1}{(q-1)\omega_{\mathrm{D}} + \mathrm{q}} = \frac{\alpha_{\mathrm{D}}}{q-1}. \tag{44}$$

Because of the presence of the deviation constant Δ_{D}, the coefficient $\widehat{\mathrm{G}}_{[q+1]}$ will not be quite the same as the effective Newtonian coupling constant $\mathrm{G}_{[q+1]}$ that will be observed in the static nonrelativistic limit for which, in terms of the relevant rest frame unit vector u^{μ} (with $u^{\mu}u_{\mu} = -1$), the stress-energy density will be approximately of the form

$$T^{\mu\nu}_{\mathrm{M}} = \rho u^{\mu}u^{\nu}, \tag{45}$$

in which ρ is the mass density, whose space volume integral will be identifiable in this limit as the total mass M. It can be seen that for a spherically symmetric distribution the time component

$$h_{00} = u^{\mu}u^{\nu}h_{\mu\nu}, \tag{46}$$

of the metric perturbation will be given in terms of the radial distance r from the center by an expression of the standard (dimensionally generalised; Arkani-Hamed *et al.*, 1999) Newtonian form

$$h_{00} = \frac{2\mathrm{G}_{[q+1]}M}{r^{q-2}}, \tag{47}$$

but with the effective gravitational coupling constant given by

$$\mathrm{G}_{[q+1]} = \widehat{\mathrm{G}}_{[q+1]}\left(1 + \frac{\Delta_{\mathrm{D}}}{\mathrm{q}-2}\right). \tag{48}$$

It can be seen that it will be related to the corresponding expression for the trace h^{ρ}_{ρ} of the metric perturbation by

$$h^{\rho}_{\rho} = \frac{2-(q+1)\Delta_{\mathrm{D}}}{q-2+\Delta_{\mathrm{D}}}h_{00}, \tag{49}$$

which is equivalent to

$$h_{00} = \frac{(\mathrm{q} - 2)\omega_{\mathrm{D}} + q - 1}{2\omega_{\mathrm{D}} + 1} h^{\rho}_{\rho}. \tag{50}$$

6. CONCLUSION

It can be seen that the ratio (50) can be matched by the simulation effect leading to the corresponding ratio (23) if the relevant reference frame curvature vector K^{μ} and the corresponding acceleration vector a^{μ} are related by

$$a^{\rho} a_{\rho} = \frac{(q - 2)\omega_{\mathrm{D}} + q - 1}{2\omega_{\mathrm{D}} + 1} K^{\rho} a_{\rho}, \tag{51}$$

or equivalently by

$$a^{\rho} K_{\rho} = \frac{2 + (\mathrm{q} + 1)\Delta_{\mathrm{D}}}{q - 2 + \Delta_{\mathrm{D}}} a^{\rho} K_{\rho}. \tag{52}$$

This means that in the linear approximation we have been using, the quasi-gravitational effect arising from the extrinsic curvature of the brane simulates what would be predicted by a Jordan–Brans–Dicke theory with ω_{D} given by what is obtained by solving (51), namely

$$\omega_{\mathrm{D}} = \frac{(\mathrm{q} - 1)a^{\rho} K_{\rho} - a^{\rho} a_{\rho}}{2a^{\nu} a_{\nu} - (q - 2)a^{\nu} K_{\nu}}, \tag{53}$$

which corresponds to a dilatonic deviation Δ_{D} given by

$$\Delta_{\mathrm{D}} = \frac{2a^{\rho} a_{\rho} + (2 - q)a^{\rho} K_{\rho}}{(q + 1)a^{\nu} a_{\nu} + a^{\nu} K_{\nu}}. \tag{54}$$

It is to be emphasized that the approximation presented here has been derived only for static configurations in a linearized weak field limit, and cannot be expected to remain accurate when stronger fields or significant deviations from staticity are involved.

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